#### Example: Representation of negative binary numbers in two’s complement

You already know how to convert binary numbers to the decimal system to determine their value as an unsigned decimal number. Below are some examples of how to interpret the values of negative binary numbers in 8-bit two’s complement:1010 01012==1·=−−128+32+4+191−−102277 +1·2+1·225+1·2+1·212+1·2+1·200

100001112=1·==−−128+4+2+1121210===1·−−128+6464−1027 +1·26

1100 0000

Binary Arithmetic 1100 11002=1·==−−128+64+8+452−1027 +1·26 +1·23 +1·22

In this section we will briefly describe how to calculate in the binary system. We will deal with the basic arithmetic operations of addition, subtraction, and multiplication. You will see that the execution of these arithmetical operations in the binary system is very simple and can therefore be realized efficiently with logic circuits, which is an enormous gain for computer science. In order not to go beyond the scope of this course, we will not consider the somewhat complicated division of binary numbers.

Addition works just like the written addition you learned in elementary school. The numbers are written on top of each other and then added up in places from right to left. The following rules apply to the addition of the individual digits:The fourth formula means that we have a carry of 00112222 +0+1+0+12222=0=1=1=1022221 at the corresponding position, which

is added to the next position.

Let us make some examples of addition:1. Let b1 := 10112 and b2 := 00112, we add the numbers from right to left.

bit 0 of the result is 0 and we note a carry of 1 for bit 1.Bit 0 is 1 for both numbers, so according to the above rule the result is 12 +12 =102, so

Bit 1 is again 1 for both numbers, the sum of which is again we also have a carry for bit 2.bit 0. The result is therefore 12 +12 +12 =102 +12 =112, so bit 1 of the result is 1 and1 +1 = 10 . In addition, however, we must also take into account the carry that resulted from the addition at2 2 2

So bit 2 of the result is 1. There is no carry to the next bit this time.Bit 2 is 0 for both numbers, but we have to add the carry bit. So it is 02 + 02 + 12 = 12.

result remains for bit 3.Bit 3 of b1 is 1, bit 3 of b2 is 0. The sum of 12 +02 = 1. Since no carry is added, this

2. So it’s Let b1 := 110110112 + 00112 and b2 = 11102 := 10012. 2 and we add the numbers again from right to left.

have a carry of 1.The addition of the bits in place 0 gives 12 +12 =102, so bit 0 of the result is 0 and we

For bit 1 the result is accordingly 02 +02 +12 = 12. There is no carry.

1Bit 2. 2 of b1 is 1, bit 2 of b2 is 0. There is no carry. Thus bit 2 of the result is 12 +02 +02 =

result is therefore 0 and another digit is added to the result which is set to 1.Bit 3 is 1 for both numbers. There is no carry. This results in 12 +12 = 102 bit 3 of the

well have more digits than the two summands.Thus, it is 11012 + 10012 = 101102, so as you can see, the result of the addition may

Let’s look at the subtraction. This works similar to addition. Here too, the numbers are subtracted from right to left and here too there are four rules:

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value 1 to the next digit, as in the case of addition, we subtract the value 1 from the next0digit in the case of a negative carry. At the same time a 1 is written to the current position2 −12 = −12 means that we can have a negative carry this time. So instead of adding the112222 −−−−01012222=0=1=0=−22212

as the result. The procedure is similar to the written subtraction of decimal numbers.

Let’s take a few examples here as well:1. Let b1 := 10112 and b2 := 01112 and subtract the numbers in places from right to left.

For bit 0 the result is 12 −12 =02. We do not have a carry.

For bit 1 the result is 12 −12 =02, again there is no carry.

For bit 2 the result is digit and note 1 as the result for the current digit.02 −12 = −12, so this time we have a carry from −1 to the next b1:

Finally, for bit 3 the result is12 −02 −12 = 02, taking the carry into account.

2. Let So the result is b1 := 010021011 and 2b − 01112 := 00112 = 01002 and subtract the numbers again from right to 2. left.

For bit 0,02 −12 = −12 results in a 1 at this position and a carry to the next position.

We have to be careful with the next bit! Bit 1 of next digit as in the previous case, but we have to note a 0 instead of a 1 as the resultin 02 −12 = −12. However, we must also consider the carry. So we have a carry to theb1 is 0, bit 1 of b2 is 1. This would result

for this bit.

−For bit 02 −12 we must again consider the carry from the previous position. The result is 2 = 02 and this time we have no carry over to the next digit. 12

Bit 3 is 0 for both numbers, so since we have no carry, the result for bit 3 is02 −02 =02.

When subtracting, please note that the above procedure only works if the first numberSo the result is 01002 − 00112 = 00012.

(the minuend) is greater than the second number (the subtrahend). If this is not the case, the subtraction must be made by adding the two’s complement of these numbers.

Next we will look at multiplication. Again, there are four calculation rules for this:The written multiplication for binary numbers works exactly as you know it from decimal00112222 ·0·1·0·12222=0=0=0=12222

numbers: You write both numbers with a ℤ linked next to each other. Then start with the last digit of the second factor and multiply it by the first factor. The result is written rightjustified under the second factor. Now multiply the penultimate digit of the second factor by the first factor and write the result under the first result, but shifted one bit to the left. The last digit is filled with a zero. This procedure is continued iteratively until all digits of the second factor have been multiplied once by the first factor. Each intermediate result is one place further to the left than the result above it. Finally, the individual results, which now stand under each other, are summed up. This gives the overall result of the multiplication.

Let us illustrate this procedure with some examples:

1. leave some space underneath for the result.Let b1 := 10102 and b2 := 10112. We write these linked by a ⋅ next to each other and 10102 ⋅ 10112

first number in sequence. The following applies: Now we first multiply the last digit of the second number (the 1) by all digits of theand . The first intermediate result is therefore12 ⋅ 02 =02,12 ⋅ 12 =12,12 ⋅ 02 =02

Now we repeat this procedure with the penultimate digit of the second number, which is also a 1. We now write the result one place shifted to the left under the first intermediate result, whereby we replace the last place with a 0 fill.

10102 ⋅ 10112

10102

101002

The next digit of the second number is 0. Accordingly, the entire interim result is 0, which we write — shifted one more digit to the left — below the previous result. We fill the last two digits with 0.

0000002

The last digit of the second number is a 1, so 10102 is the intermediate result. Again, we write this under the previous result, moving one digit further to the left and filling the last three digits with 0.

0000002

It results in:

Again, we add the individual results and get by to add a 0 as the last digit to the positive binary number.102 =210. In such a case, the calculation is extremely simple because you only have1101 ⋅ 0010 = 11010 . This example illustrates a nice special case, namely the multiplication of a positive binary number2 2 2